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Vision Research 46 (2006) 2311–2314

Vision Research

www.elsevier.com/locate/visres

Brief communication

Role of a circle's center in visual interpolation ☆

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Received 18 December 2004; received in revised form 22 November 2005

Abstract

Basic geometric patterns like straight lines and circles seem fundamental to human perception and mental imagery. In this study we examined subjects' ability to interpolate circular curves—to derive the whole circle from an arc of 180° or less. Specifically, we tested how the center point is utilized during such visual interpolation. Naturally, a mechanism that interpolates by extending the curvature of the visible arc will be unaffected by the presence or absence of the center point. On the other hand, a mechanism that achieves the same end by completing the circle from estimates of the center and radius will be significantly aided by the presence of the center. We found that when the visible arc was long (180°), presenting the circle's center did not affect the precision with which subjects localized the invisible section. However, when the visible arc was relatively short (90° or 45°), displaying the center point significantly increased spatial precision. Thus, both computational mechanisms appear to exist.

Keywords: Mental computation; Circular curve; Visual interpolation

1. Introduction

One basic function of human vision is to encode information about various shapes. Previous studies have investigated several important aspects of the underlying mechanisms. One line of research has focused on global configural information for the purpose of object classification and recognition (e.g., Biederman, 1987; Marr & Nishihara, 1978); Another line of research has focused on describing the subtle feature discriminations that observers can make (e.g., Burbeck & Pizer, 1995).

Basic geometric elements like straight lines and circles are easily, and seemingly intuitively, apprehended by humans. Studying how people make judgments about them may help us understand some fundamental aspects of cognition and perception. Straight lines have been a topic of much investigation (e.g., Ludvigh, 1953, for hyperacuity in 3-dot alignment; see also Tyler, 2002 for another example of a special geometric relation: symmetry). Perception of circles has also been studied under the rubric of visual interpolation. Such research has asked the question: when only part of a circle is visible, how do people determine the position of the invisible part (Guttman & Kellman, 2004; Schoumans & Sittig, 2000; Takeichi, 1995).

Previous research on the visual interpolation of circles has not investigated the role that the center of the interpolated circle plays during the process of interpolation. In the present study, we intend to tease apart the contributions of two mechanisms of circle completion by capitalizing on the fact that different computational strategies would be differentially affected by the display of the exact center of the circle. First, subjects could complete the invisible section of the circle by computing the local curvature $\boldsymbol{k}$ of the visible arc and then extending the arc with curvature fixed at $\boldsymbol{k}$. We will call this *curvature continuation*. Such a mechanism is implied in previous work on visual interpolation (e.g.,

☆ This research was supported by Grant MH45584 from the National Institute of Mental Health to Harold Pashler.

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0042-6989/$ - see front matter
doi:10.1016/j.visres.2006.01.031

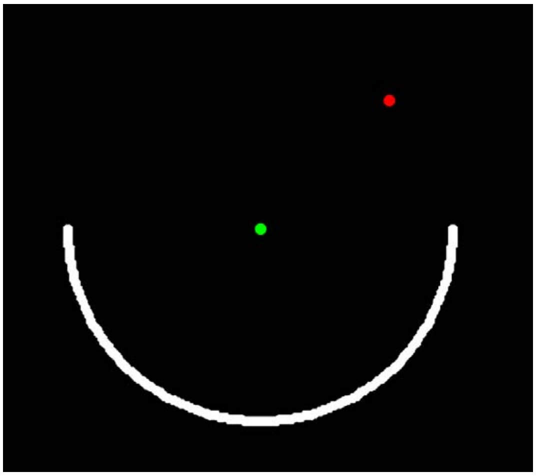

Fig. 1. A sample display. Each display consisted of one white arc and one red test dot on a black screen. Subjects were asked to determine whether the test dot was inside or outside the implied circle. For half of the trials, the center of the circle was marked with a green dot. (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of this article.)

Takeichi, 1995), but is usually not explicitly stated. Second, subjects could complete the invisible section of the circle by computing distance from the circle's center: all points at distance ***r*** (radius) from the center ***o*** would thus be selected. We will call this mechanism *radius computation*. The basic idea behind this mechanism is analogous to that outlined in Medial-point descriptions of shape (Kovacs & Julesz, 1993, 1994; Kovacs, Feher, & Julesz, 1998; see also Van Tonder, Lyons, & Ejima, 2002).

Curvature continuation and radius computation make different predictions about subjects' performance. If a circle is completed by computing distance from the center, then the center itself must first be calculated from the visible arc. This step will naturally need some computation and introduce some level of noise. Therefore, if the center point is displayed, this step will not be necessary, and the circle should be completed more accurately. On the other hand, if a circle is completed by curvature continuation, then displaying the center of the circle should have no effect on the spatial precision of the completed section.

Subjects were briefly presented part of a circle and one dot (Fig. 1), and were asked to determine whether the dot was inside or outside the implied circle. We measured the threshold at which subjects could accurately make this judgment in terms of the distance from the test dot to the closest point on the perimeter of the circle, if the circle were complete. This measure effectively described the precision with which subjects completed the invisible section of the circle.

We measured this threshold as we manipulated three parameters. The *length of the visible arc* was manipulated across subjects. Within subjects we varied the *angular distance (degrees) between the test dot and the visible arc*, and *whether or not the center point was displayed.*

2. Method

2.1. Subjects

University undergraduates participated in this experiment for course credit. All subjects had normal or corrected-to-normal vision. The 118 subjects were assigned to one of three visible arc length conditions (180°: 52 subjects; 90°, 29 subjects and 45°, 37 subjects).

2.2. Apparatus

Stimuli were presented on 1024 × 768 color monitors. Subjects viewed the displays from a distance of about 60 cm and entered responses on a standard keyboard. The experiment was programmed in Microsoft Visual Basic 6.0 and run on Microsoft Windows 98, second edition.

2.3. Stimuli

As shown in Fig. 1, each display contained one white arc on a black screen. The length of the arc was 180°, 90°, or 45°. The length was manipulated across subjects, so for each subject the visible arc length remained constant during the entire experiment. The visible arc was always part of the lower half of the circle. The circle was always presented at the center of the display, but with a jitter of ±1 cm both horizontally and vertically. The radius of the circle was 2.8 cm, and the width of the line illustrating the arc was 0.14 cm. A red test dot was displayed along the invisible section of the circle in one of seven angular positions. The radial displacement (inward or outward) of the test dot from the perimeter of the circle was varied in a staircase procedure (described in Procedure). For each of the three visible arc length conditions, the seven possible angular positions of the test dot were evenly spaced in the "invisible" section of the circle at 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, and 7/8 of the full range (e.g., if the visible arc extended from 0° to 90°, the seven possible angular positions of the test dot would be 123.75°, 157.5°, 191.25°, 225°, 258.75°, 292.5°, and 326.25°). Subjects were asked to determine whether the red dot fell inside or outside the implied circle. In half the trials, the center of the circle was marked with a green dot. The presence (or absence) of the green center dot and the position of the red test dot varied from trial to trial.

2.4. Procedure

Subjects were asked to respond as accurately as possible. Each trial began with a small fixation cross presented for 300 ms at the center of the screen. After a 300 ms blank interval, the task stimuli were presented for 300 ms. Subjects were instructed to fixate the cross, and then to determine whether the red test dot fell inside or outside of the circle implied by the visible arc. Subjects responded by pressing one of two adjacent keys ('j' for *inside;* 'k' for *outside*) and were provided with feedback (a tone) to indicate whether the response was correct. The next trial began 300 ms later. Each subject performed eight blocks of 140 trials each. The first two blocks were considered practice and were excluded from analysis.

The radial displacement of the test dot from the perimeter of the circle was controlled by a standard staircase procedure. Two consecutive correct responses lead to a 1 dB decrease, and one incorrect response lead to a 1 dB increase. A threshold was computed for each subject as the average radial displacement (on a logarithmic scale) of all of the trials in each of the 14 conditions (center dot present or absent, by seven angular positions).

3. Results

Results for each of the three visible arc length conditions are plotted in Fig. 2. When the visible arc length was 180°, presentation of the center dot had no significant impact ($F(1, 51) = 0.36$); but it was significantly helpful when the

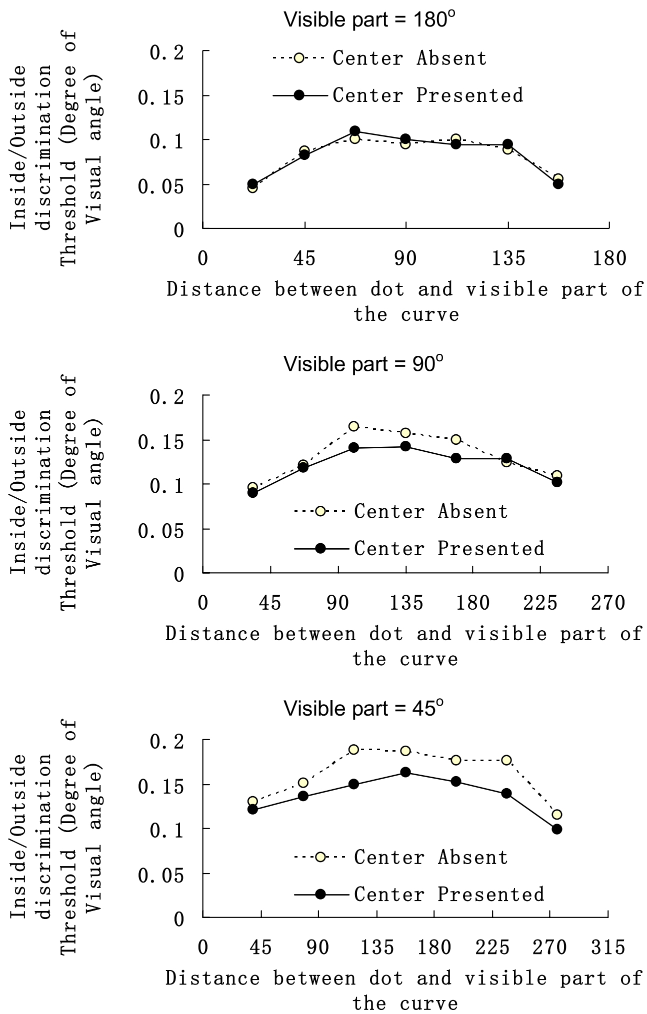

Fig. 2. Discrimination thresholds (degrees of visual angle) for three visible arc lengths as a function of angular distance from the test dot to the visible arc. Presenting the center dot had no effect with an arc length of 180° but was significantly helpful when arc length was 90° or 45°.

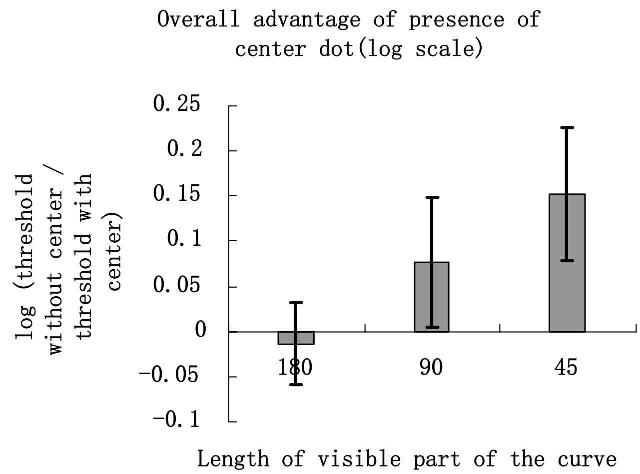

Fig. 3. Magnitude of the benefit (log (threshold without center/threshold with center)) of displaying the center dot for the three visible arc length conditions. This benefit is negligible for 180° arcs but is significant for 90° and 45° arcs.

arc length was 90° ($F(1,28) = 4.84$; $p < 0.05$) and 45° ($F(1,36) = 17.69$; $p < 0.00025$). Fig. 3 shows the increase in precision from presenting the center dot for all 3 visible arc length conditions (logarithm scale). The magnitude of the advantage of presenting the center dot increased as visible arc length decreased ($F(2,115) = 8.53$, $p < 0.0005$).

4. Discussion

Our results suggest that subjects can precisely complete an entire circle from a 180° arc alone: displaying the center of the circle does not help complete the invisible portion. Therefore, curvature continuation must be functionally important. However, when the visible arc is shorter (45° or 90°), displaying the center of the circle helps significantly. Thus, it seems that radius computation is also functionally important.

It is plausible that a longer visible arc might allow the local curvature to be calculated more quickly and precisely. Furthermore, an initial imprecision in estimating curvature will result in a greater loss of precision as it is used to complete the curve over long distances. Therefore, curvature continuation should be relatively accurate over short distances but should become increasingly less accurate as distance from the visible arc increases. For both reasons (less accurate initial curvature estimation and longer distance of curve completion) the overall precision of curvature continuation will drop when the length of visible arc is decreased. On the other hand, precision of radius computation probably depends much less on visible arc length. Thus, it is likely that curvature continuation is used to complete the circle when the visible arc is long because it is more precise than radius computation under those conditions. However, when the visible arc is relatively short, curvature continuation becomes less precise, and radius computation is used instead.

Previous research on visual interpolation demonstrated the remarkable ability of the human visual system to fill in missing circular curves (e.g., Takeichi, 1995). Many factors affecting the interpolation of such curves have been studied (e.g., closure effect in fragment connection: Kovacs & Julesz, 1993; convexity and concavity: Fantoni, Bertamini, & Gerbino, 2005; amount of available arc: Guttman, Sekuler, & Kellman, 2003; see Kovacs, 1996 for a review). However, to our knowledge, no previous research has addressed how and under what conditions the center of a circle is used to interpolate a missing section of the circle. By manipulating whether a circle's center point was displayed, we could differentiate two distinct computational strategies that are utilized in circle completion: radius computation and curvature continuation. Radius computation is a strategy similar to the mechanisms often used in other tasks to compare distance and size. Curvature continuation seems to be a different mechanism that can be employed for calculating other geometric relations (like straight lines).

Because perception and mental creation of circles both seem so natural, it is easy to overlook the necessity and significance of this specialized mechanism. For comparison, consider another mathematically simple and functionally useful curve: the parabola. Like circles, parabolas are computationally simple and are common in daily life (e.g., the flight of a ball). The ability to accurately complete parabolas (e.g., to visualize the complete trajectory of a tossed object) would be very useful. Nonetheless, humans seem not to have developed mechanisms for this purpose: it is very difficult for us to complete a parabola from one visible subsection.

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